HHIF Lecture Series: Risk

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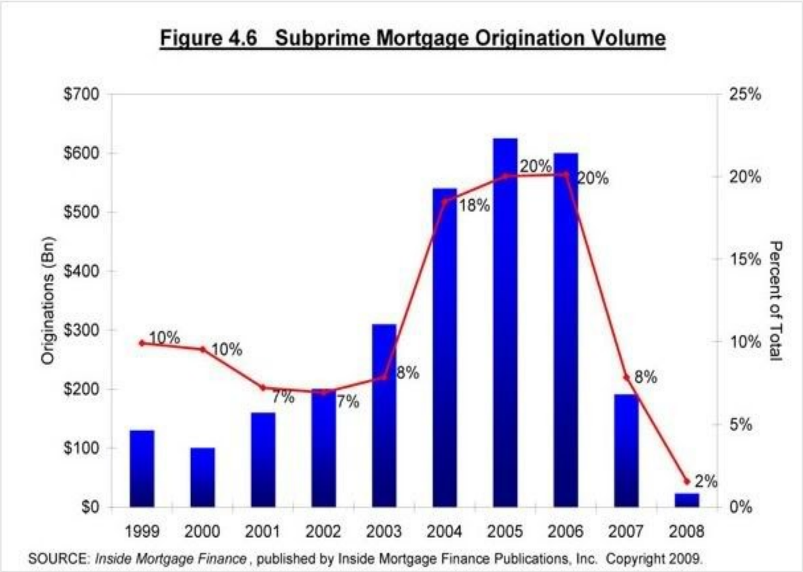
February 4, 2011

What is Risk?

- Risk measures the degree of **uncertainty**. When it comes to investing, this uncertainty is about the **actual** return on an asset.
- Every investment is associated with an expected return and risk
- Another way to define an undervalued asset: provides a return higher than the return required to compensate for risk
- Different investors are willing to be exposed to different amounts and types of risk

- The financial meltdown of 2007-2008 occurred because of insufficient risk management practices across the board
- Risk Management is a very important area and is guaranteed to be in strong demand in the future

US Sub-Prime Mortgage Origination



Different Types of Risk borne by Investors

- **Market (systematic) Risk**: arises from exposure to the whole market
  - Almost impossible to eliminate
- **Unique (unsystematic) Risk**: specific to the investment
  - Can be eliminated through diversification
- **Interest Rate Risk**: arises from fluctuations in interest rates
- **Exchange Rate Risk**: arises from holding investments denominated in other currencies
- **Credit Risk**: risk that the security issuer will not settle the obligation due to default
- **Liquidity Risk**: The security cannot be converted to cash without a large loss due to lack of demand
- **Technology Risk**: The trading system may fail (e.g. Flash Crash)
- **Operational Risk**: Risk due to inadequate internal risk controls in a company

Various Securities and Risks Associates with Them

- **Money Market** - short term debt (matures in less than 1 year)
  - e.g. T-bills, commercial paper
  - Risks: **inflation**, interest rate, unique

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  - e.g. Options, Futures, Swaps, Warrant
  - **Leverage** - need to put up only a small portion to get exposure to the asset
  - Potential for huge returns, but high chance to lose everything
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- There is also foreign exchange risk borne if investing in securities in another country

Human Perception of Risk - Risk Aversion

- **Risk Aversion** - reluctance of being exposed to risk
  - For example, a risk averse investor will prefer an investment offering a guaranteed 5% return over an investment offering an expected return of 6% but with the potential to return only 1%
  - An investor is *risk-neutral* if he is indifferent about the amount of risk he undertakes as long as he is offered the same expected payoff
  - A *risk-loving* investor will prefer an investment offering a higher level of risk but a lower expected return
- **Expected Utility Theory** - the objective of the investor is to maximize expected value of utility, some function of terminal wealth
  - However, this function depends on the personal objectives and risk profile of investor
  - Still unclear what is the best way to model investor behavior - **behavioral finance**
  - Recent developments of prospect theory, started by Kahneman and Tversky (1979), measures losses and gains
  - People tend to overestimate low probabilities and underestimate high probabilities

Measuring Risk - Standard Deviation

- We will focus on the first two types of risk - market and unique risk
- Usually measured as the standard deviation (volatility) of returns

$$E(R) = \sum_s p(s)R(s) \text{ - expected return}$$

$$\sigma(R) = \sqrt{\sum_s p(s)(R - E(R))^2} \text{ - standard deviation,}$$

where s are states of the world, and R is return

- Consider paying \$1 to flip a fair coin once. If the coin lands heads up, you win \$1; if it lands tails up, you lose \$0.5.
  - The expected return is $\frac{1}{2}100\% + \frac{1}{2}(-50\%) = 25\%$
  - The standard deviation is $\sqrt{\frac{1}{2}(100-25)^2 + \frac{1}{2}(50-25)^2} \approx 55.9\%$
- However, for real life investments, it is not so easy
  - There are a huge number of possible events corresponding to different future returns
  - Need sophisticated tools to estimate the probability of each event (e.g. Monte Carlo simulation)
  - Or can use historical returns to get standard deviation

Measuring Risk - Scenario Analysis

Example: Research and Development

- **Scenario Analysis** - used in risk-return analysis of business projects
- A tech firm considers investing \$20MM to develop a new product.
- Consider two variables:
  - results of development and results of R&D efforts of a competitor
- For each scenario, estimate amount of profit generated from the new product and the probability that the scenario happens

Scenario Analysis Example				
Firm Results	Competitor Results	Profit	Return	Probability
Very Effective	Effective	\$26MM	30%	$0.3 \times 0.7 = 0.21$
	Not Effective	\$40MM	100%	$0.3 \times 0.3 = 0.09$
Effective	Effective	\$23MM	15%	$0.4 \times 0.7 = 0.28$
	Not Effective	\$31MM	55%	$0.4 \times 0.3 = 0.12$
Not Effective	Effective	\$15MM	-25%	$0.3 \times 0.7 = 0.21$
	Not Effective	\$21MM	5%	$0.3 \times 0.3 = 0.09$

Measuring Risk - Historical Data

- Consider returns $r_1, r_2, \ldots, r_n$ in n periods

$$\bar{r} = \frac{r_1 + r_2 + \ldots + r_n}{n} \text{ - estimate of expected return}$$

$$\sigma = \sqrt{\frac{(r_1 - \bar{r})^2 + \ldots + (r_n - \bar{r})^2}{n-1}} \text{ - estimate of standard deviation}$$

- For example, consider an investment that returned 10%, -7%, 4%, 11%, 3% in the past five years.
  - Estimate of expected return: $\frac{10-7+4+11+3}{5} = 4.2\%$
  - Estimate of standard deviation: $\sqrt{\frac{5.8^2+11.2^2+0.2^2+6.8^2+1.2^2}{4}} \approx 7.19\%$
  - If we divide by 5, the estimate is 6.43%
- If you use historical data to estimate standard deviation, use a lot of periods
- Standard deviation of a security varies through time...
  - But it is fast and simple

Diversification - I

- **Covariance**: used to measure how a change in one random variable is related to the change in another

$$cov(R_1, R_2) = \sum_s p(s)(R_1(s) - E(R_1))(R_2(s) - E(R_2));$$

$$\rho(R_1, R_2) = \frac{cov(R_1, R_2)}{\sigma(R_1)\sigma(R_2)} \text{ - correlation}$$

where R_1, R_2 are returns on two assets A_1, A_2

- Can also estimate correlation and covariance based on historical data
- Correlation is always between -1 and 1
  - Close to 1: returns are (positively) correlated, i.e. move together
  - Close to -1: negatively correlated, tend to move in different directions
  - Close to 0: changes in returns don't seem related
- Correlation between assets changes with time
- High correlation does not imply a cause-and-effect relationship

Diversification - II

- Consider a portfolio P of n assets $A_1, A_2, \dots, A_n$
  - Asset A_i provides return R_i and has weight w_i in portfolio
  - Interested in return R on the whole portfolio
  - Enough to know expected values, standard deviations, and correlations

$$E(R) = w_1 E(R_1) + w_2 E(R_2) + \dots + w_n E(R_n);$$
$$\sigma^2(R) = \sum_{i=1}^{n} w_i^2 \sigma^2(R_i) + \sum_{i<j} 2 w_i w_j \sigma_{ij};$$
$$\text{where } \sigma_{ij} = cov(R_i, R_j) = \rho(R_i, R_j)\sigma(R_i)\sigma(R_j)$$

- **Benefits of Diversification**:
  - If the assets in the portfolio are not perfectly positively correlated (i.e. $\sigma_{ij} = 1$), the standard deviation of the portfolio is less than the weighted sum of the standard deviations
  - Heuristically: a loss in some assets will be offset by gains in other assets
  - Extreme Case: two assets A_1, A_2 and $\sigma_{ij} = -1$; then $\sigma(R) = \sqrt{w_1^2 \sigma^2(R_1) + w_2^2 \sigma^2(R_2) - 2 w_1 w_2 \sigma(R_1)\sigma(R_2)} = |w_1 \sigma(R_1) - w_2 \sigma(R_2)|$

Diversification - Example

Portfolio of 3 Assets

- Consider a portfolio made up of 60% in an equity fund, 30% in a bond fund and 10% in a money market fund
  - **Equity Fund**: expected return 16%, standard deviation 13%
  - **Bond Fund**: expected return 6%, standard deviation 7%
  - **Money Market Fund**: expected return 2%, standard deviation 0.5%

Correlation Matrix

	Equity	Bond	Money Market
Equity	1	0.3	0.07
Bond		1	0.1
Money Market			1

- Then for the portfolio:
  - $E(R) = 0.6 \cdot 16\% + 0.3 \cdot 6\% + 0.1 \cdot 2\% = 11.6\%$
  - $\sigma(R) = \sqrt{0.6^2 \cdot 13^2 + 0.3^2 \cdot 7^2 + 0.1^2 \cdot 0.05^2 + \text{ corr. terms}} \approx 8.67\%$
  - The portfolio has expected return 11.6% and standard deviation 8.67%

Diversification - Graphical Example

Two Uncorrelated Assets

- Consider a portfolio of 2 uncorrelated assets
- Return, standard deviation of daily return - based on 2010 data
  - Bank of Nova Scotia: return 26.7%, standard deviation 1.57%
  - DollarTree: return 74.6%, standard deviation 1.48%
  - Correlation between daily returns: 0.145
  - Portfolio of 7 DLTR, 1 BNS shares: return 66.6%, std. dev. 1.39%
  - Portfolio of 1 DLTR, 1 BNS share: return 46.7%, std. dev. 1.14%
- Caution: this is does not mean the trend will continue in the future
  - But we still expect BNS and DLTR to have low correlation in the next few years
- More stocks in portfolio - more potential for diversification
  - Different levels of diversification
    - Security Type, Country, Sector, Sub-Sector, Stock
  - But with a highly diversified portfolio, do not expect a great return

Diversification - Graphical Example

Two Uncorrelated Assets

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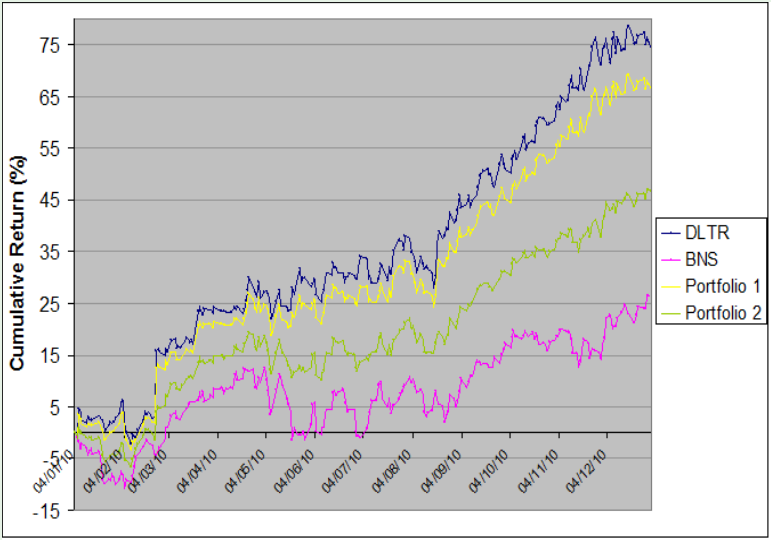


Portfolio with Risk-Free Asset

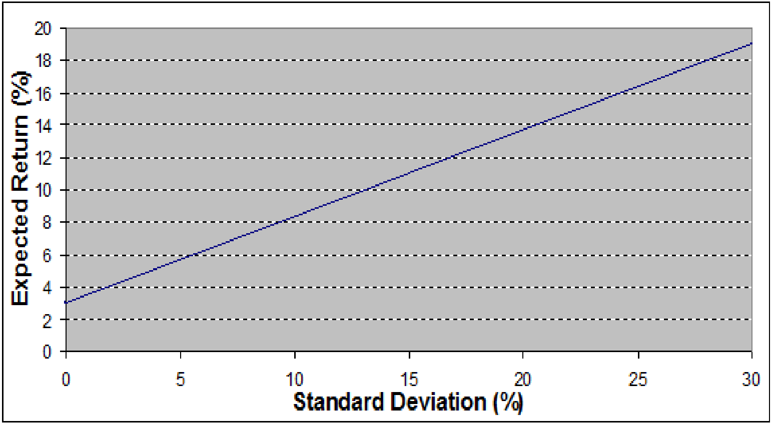
- Consider a portfolio P made up of 2 assets A and B
  - A is a "**risk-free**" asset: expected return r_f, standard deviation **0%**
  - Risk-free asset uncorrelated with any other asset
  - B is a risky asset, expected return r, standard deviation σ
  - Risk-free asset does not exist in real life, everything has some risk
  - Usually US T-bill is considered risk-free, is that really so?
- Let ω is weight of asset B, $1-\omega$ be the weight of asset A in portfolio
  - $1-\omega > 0$: *lending*, $1-\omega < 0$: *borrowing*
  - Assume for now borrowing and lending rates are the same
- Portfolio Expected Return: $E(R) = (1-\omega)r_f + \omega r = r_f + \omega(r - r_f)$
  - $r - r_f$ is called *risk premium*
- Standard Deviation:

$$\sigma(R) = \sqrt{(1-\omega)^2 0^2 + \omega^2\sigma^2 + 2(1-\omega)\omega r_f r \cdot 0} = \omega\sigma$$

$$\boxed{E(R) = r_f + \omega(r - r_f), \sigma(R) = \omega\sigma}$$

Capital Allocation Line

- Assume investor can only invest in assets A and B
- Based on ω, we can find expected return and standard deviation
- All possible expected return-standard deviation points can be plotted
- They form a line, called the *capital allocation line*



Maximizing Investor Utility

- Assume investor has the following utility funcion:
  $U = E(R) - \frac{1}{2}k\sigma^2(R)$
  - k is a risk aversion coefficient, some positive number
  - Expected return and Standard Deiviation correspond to same period
- Can only invest in assets A and B and wants to maximize utility
- Problem: Find ω to maximize
  $U = E(R) - \frac{1}{2}k\sigma^2(R) = r_f + \omega(r - f_f) - \frac{1}{2}k\omega^2\sigma^2$
  Differentiate with respect to ω:
  $(r - f_f) - \omega(k\sigma^2) = 0$
  $\boxed{\omega^* = \dfrac{r - r_f}{k\sigma^2}}$ - optimal weight of risky asset in portfolio
- In real life things are more complicated
  - Risk Aversion changes (age, personal circumstances, etc.)
  - Investment Objectives change
  - Need to regularly review what the weights should be
  - This is where investment advisors come in

Concluding Remarks

- Think not only about return, but also about risk
- Standard Deviation (volatility) - common measure of risk
- Can estimate standard deviation using historical returns
  - However, it changes with time
- If you know correlations between assets in portfolio, can estimate risk
  - Holding uncorrelated assets usually results in lower risk
  - However, it also results in lower return
- A typical investor is risk-averse - does not like risk
  - Expects higher return if exposed to more risk
  - Different investors have different risk profiles

- To understand how these concepts work in real life:
  - Select several stocks
  - Download historical data from Yahoo Finance into Excel
  - Using Excel find returns, stdandard deviations, correlations
  - Create arbitrary portfolio weights
  - Find portfolio risk for the period

Any Questions?

Upcoming Events:

- Next Lecture: Modern Portfolio Theory and Asset Pricing
  - Friday, February 18, 6:00-8:00 p.m., South Dining Room, Hart House

References

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